8/7/24 (FN)

Reg. No. | | | | | | | | | | |

ANNA UNIVERSITY (UNIVERSITY DEPARTMENTS)
B.E./B.TECH (FULL TIME), END SEMESTER EXAMINATIONS, APRIL/MAY 2024
COMMON TO ALL BRANCHES
FIRST SEMESTER

MA3151 – MATRICES AND CALCULUS
(REGULATIONS 2023)

Duration: 3 hours **Maximum Marks: 100**

Answer "All the Questions"
Part-A ($10\times 2 = 20$ Marks)

Q. No.	Questions	Marks
1	Write down a matrix A of order 3x3 such that the eigen values of the matrix A^3+I are 1947, 1950 and 2024.	2
2	Does the identity matrix of order 3x3 have an eigen vector with length 2024? Justify your answer.	2
3	The diameter and altitude of a can in the shape of a right circular cylinder are measured as 4 cm and 6 cm respectively. The possible error in each is 0.1 cm. Find approximately the maximum possible error in the value computed for the lateral surface.	2
4	If $u = f\left(\frac{x^2+2xy}{y^2-3xy}, \frac{y^2-4xy}{x^2+5xy}\right)$, then find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.	2
5	Find the Cauchy's principal value of the improper integral $\int_{-\infty}^{\infty} \sin\left[\sin\left(x^{N^2-N+1947}\right)\right]dx$ where N is the year in which your mathematics teacher was born.	2
6	Find the value of $\Gamma(-1/2)$.	2
7	Evaluate $\int_0^1\int_0^{1-x} xy\,dy\,dx$.	2
8	Using double integration, find the area of the circle $r=1$.	2
9	Find the work done in moving a particle in the force field $\vec{F} = 3xy\vec{i} - y^2\vec{j}$ in the xy-plane along the curve $y=x$ from the point (0, 0) to (1,1).	2
10	Define the notion of directional derivate of a scalar valued function $\phi(x,y,z)$.	2

Part- B ($5\times13=65$ Marks)

Q. No.		Questions	Marks
11 a	i	Find all the eigen values of the matrix $A^2+2A-3I$ if $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ where $a+c=1,\ a+d=1,\ b+d=1$. (*Your answer should NOT be in terms of a, b, c, d.*)	6
	ii	Verify Cayley-Hamilton theorem for the matrix $A=\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and hence find its inverse.	7
		OR	
11 b		Reduce the matrix $A=\begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$ to the diagonal form by using similarity transformation.	13
12 a	i	Using Taylor's theorem, expand $f(x,y)=e^x\log(1+y)$.	6
	ii	If $z=f(x,y)$ and $x=e^u\cos v$ and $y=e^u\sin v$, then prove that $x\frac{\partial z}{\partial v}+y\frac{\partial z}{\partial u}=e^{2u}\frac{\partial z}{\partial y}$ and $\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2=e^{-2u}\left[\left(\frac{\partial z}{\partial u}\right)^2+\left(\frac{\partial z}{\partial v}\right)^2\right]$.	7
		OR	
12 b	i	If $u=f(x-y,y-z,z-x)$, then show that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$.	6

	ii	Find the minimum and maximum values of $f(x,y)=x^4+y^4-2x^2+4xy-2y^2$.	7
13 a	i	Show that $\int_a^b (x-a)^{m-1}(b-x)^{n-1}\,dx=(b-a)^{m+n-1}\beta(m,n)$.	6
	ii	Using the concept of differentiation under integral sign, evaluate $\int_0^a \frac{\log(1+ax)}{1+x^2}dx$.	7
OR			
13 b	i	Show that $\beta(m,n)=\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.	6
	ii	Evaluate $\int_0^\infty xe^{-x^8}dx \times \int_0^\infty x^2e^{-x^4}dx$.	7
14 a	i	Find the area that lies inside the cardioid $r=1+\cos\theta$ and outside the circle $r=1$ by using double integration.	6
	ii	Evaluate $\iiint \frac{dxdydz}{\sqrt{1-x^2-y^2-z^2}}$, taken over the volume of the sphere $x^2+y^2+z^2=1$.	7
OR			
14 b	i	By changing the order, evaluate $\int_0^1\int_{e^x}^{e} \frac{1}{\log(y)}dydx$.	6
	ii	By changing to polar coordinates, evaluate $\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}dxdy$.	7

15 a	i	Verify Green's theorem for $$\int_C (xy+y^2)dx + x^2 dy$$ where C is bounded by $y=x$ and $y=x^2$.	6
	ii	Verify Stoke's theorem for the vector field $$\vec{F} = (2x-y)\vec{i} - yz^2\vec{j} - y^2 z\vec{k}$$ over the upper half surface of $x^2+y^2+z^2=1$, bounded by its projection on the xy-plane.	7
OR			
15 b		Verify Gauss divergence theorem for $$\vec{F} = (x^2-yz)\vec{i} + (y^2-zx)\vec{j} + (z^2-xy)\vec{k}$$ taken over the cube formed by the planes $x=0$, $y=0$, $z=0$, $x=1$, $y=1$, and $z=1$.	13

Part- C (Compulsory Question) ($1 \times 15 = 15$ Marks)

Q. No.		Questions	Marks
16	i	Find the area bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$ by using double integration.	7
	ii	Find the dimension of the rectangular box, open at the top, of maximum capacity, whose surface area is 108 square cm.	8

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